BASIC ELECTRICAL AND ELECTRONICS ENGINEERING
LECTURE NOTES

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

MALLA REDDY ENGINEERING COLLEGE AUTONOMOUS

MAISAMMAGUDA, DULAPALLY– 500014, Hyderabad

SYLLABUS:

MODULE I : DC Circuits [09 Periods]

Electrical circuit elements (R, L and C), voltage and current sources, Kirchhoff's current and voltage laws - Series, parallel, series-parallel, star-to-delta and delta-to-star transformation-analysis of simple circuits with dc excitation. Superposition, Thevenin's and Maximum Power Transfer Theorems with DC excitation.

MODULE II: AC Circuits 09 Periods]

Representation of sinusoidal waveforms, peak and rms values, phasor representation,real power, reactive power, apparent power, power factor. Analysis of single-phase ac circuits consisting of R, L, C, RL, RC, RLC combinations (series and parallel).

MODULE III: Introduction to Electrical Machines [10 Periods]

A: DC Machines : Construction & Principle of Operation of DC Generators – E.M.F Equation. Principle of operation DC Motors – Back E.M.F. - Torque equation – Brake Test -Characteristics.
B: AC Machines: Construction and Principle of operation of Transformer- EMF Equation. Construction and Principle of Operation of 3 Phase Induction Motors - Brake test on 3-Phase Induction Motor – Applications

MODULE IV: P-N Junction Diode [10 Periods]

A: P-N Junction Diode: Diode equation, Energy Band diagram, Volt-Ampere characteristics, Temperature dependence, Ideal versus practical, Static and dynami resistances, Equivalent circuit, Diffusion and Transition Capacitances. Zener diode operation, Zener diode as voltage regulator.
B: Rectifiers : P-N junction as a rectifier - Half Wave Rectifier, Ripple Factor – Full Wave Rectifier, Bridge Rectifier.
C: Filters : Filters – Inductor Filters, Capacitor Filters, L- section Filters, π- section Filters.

MODULE V : BJT and Junction Field Effect Transistor (JFET) [10 Periods]

A:Bipolar Junction Transistor (BJT): Construction, Principle of Operation,Symbol, Amplifying Action, Common Emitter, Common Base and CommonCollector configurations and Input-Output Characteristics, Comparison of CE, CB and CC configurations

B: Junction Field Effect Transistor and MOSFET: Construction, Principle ofOperation, Symbol, Pinch-Off Voltage, Volt-Ampere Characteristic, Comparisonof BJT and FET.

TEXT BOOKS

1. M.Surya Kalavathi, Ramana Pilla, Ch. Srinivasa Rao, Gulinindala Suresh,"Basic Electrical and Electronics Engineering", S.Chand and CompanyLimited, New Delhi, 1st Edition, 2017.
2. R.L.Boylestad and Louis Nashlesky, "Electronic Devices & Circuit Theory",Pearson Education, 2007.

REFERENCES

1. V.K. Mehtha and Rohit Mehta, "Principles of Electrical Engineering andElectronics", S.Chand & Co., 2009.
2. Jacob Milliman, Christos C .Halkias, Satyabrata Jit (2011), "ElectronicDevices and Circuits", 3 rd edition, Tata McGraw Hill, New Delhi.
3. Thomas L. Floyd and R. P. Jain, "Digital Fundamentals", Pearson Education,2009.
4. David A. Bell, "Electronic Devices and Circuits", Oxford University Press,2008.

UNIT – I
DIRECT CURRENT CIRCUITS

1.1 INTRODUCTION

Given an electrical network, the network analysis involves various methods. The process of finding the network variables namely the voltage and currents in various parts of the circuit is known as network analysis. Before we carry out actual analysis it is very much essential to thoroughly understand the various terms associated with the network. In this chapter we shall begin with the definition and understanding in detail some of the commonly used terms. The basic laws such as Ohm’s law, KCL and KVL, those can be used to analyse a given network Analysis becomes easier if we can simplify the given network. We will be discussing various techniques, which involve combining series and parallel connections of R, L and C elements.

1.2 SYSTEMS OF UNITS

As engineers, we deal with measurable quantities. Our measurement must be communicated in standard language that virtually all professionals can understand irrespective of the country. Such an international measurement language is the International System of Units (SI). In this system, there are six principal units from which the units of all other physical quantities can be derived.

Quantity	Basic Unit	Symbol
Length	Meter	M
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Temperature	Kelvin	K
Luminous intensity	candela	Cd

One great advantage of SI unit is that it uses prefixes based on the power of 10 to relate larger and smaller units to the basic unit.

Multiplier	Prefix	Symbol
10^{12}	Tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	K
10^{-3}	milli	m
10^{-6}	micro	
10^{-9}	nano	n
10^{-12}	pico	p

1.3 BASIC CONCEPTS AND DEFINITIONS

1.3.1 CHARGE

The most basic quantity in an electric circuit is the electric charge. We all experience the effect of electric charge when we try to remove our wool sweater and have it stick to our body or walk across a carpet and receive a shock.

Charge is an electrical property of the atomic particles of which matter consists, measured in coulombs (C). Charge, positive or negative, is denoted by the letter q or Q.

We know from elementary physics that all matter is made of fundamental building blocks known as atoms and that each atom consists of electrons, protons, and neutrons. We also know that the charge ‘e’ on an electron is negative and equal in magnitude to 1.602×10^{-19} C, while a proton carries a positive charge of the same magnitude as the electron and the neutron has no charge. The presence of equal numbers of protons and electrons leaves an atom neutrally charged.

1.3.2 CURRENT

Current can be defined as the motion of charge through a conducting material, measured in Ampere (A). Electric current, is denoted by the letter i or I.

The unit of current is the ampere abbreviated as (A) and corresponds to the quantity of total charge that passes through an arbitrary cross section of a conducting material per unit second. Mathematically,

$$I = \frac{Q^{-}}{t} \text{or} Q = It$$

Where is the symbol of charge measured in Coulombs (C), I is the current in amperes (A) and t is the time in second (s).

The current can also be defined as the rate of charge passing through a point in an electric circuit. Mathematically,

$$i = \frac{\overline{d}q}{dt}$$

The charge transferred between time t_1 and t_2 is obtained as

$$q = \int_{t_1}^{t_2} i dt$$

A constant current (also known as a direct current or DC) is denoted by symbol I whereas a time-varying current (also known as alternating current or AC) is represented by the symbol or *I or i(t)* . Figure 1.1 shows direct current and alternating current.

Current is always measured through a circuit element as shown in Fig. 1.1

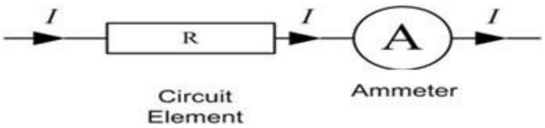


Fig. 1.1 Current through Resistor (R)

Two types of currents:

1) A direct current (DC) is a current that remains constant with time.
2) An alternating current (AC) is a current that varies with time.

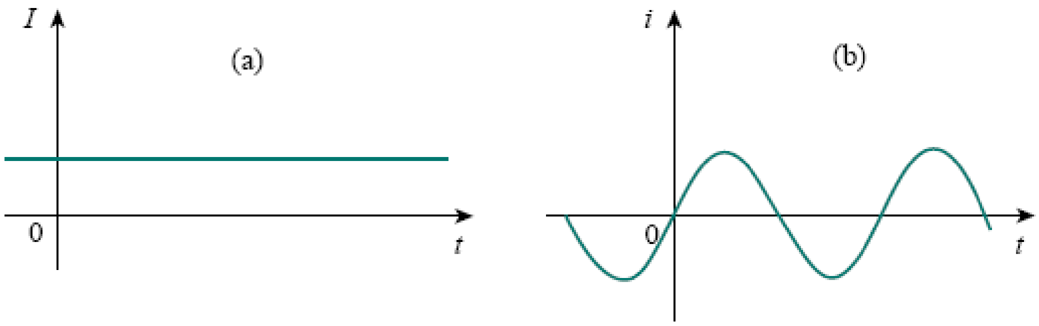


Fig. 1.2Two common types of current: (a) direct current (DC), (b) alternative current (AC)

Example 1.1
Determine the current in a circuit if a charge of 80 coulombs passes a given point in 20 seconds (s).

Solution:

$$I = \frac{\overline{Q}}{t} = \frac{\overline{80}}{20} = 4\ A$$

Example 1.2
How much charge is represented by 4,600 electrons?

Solution:
Each electron has - $1.602\text{x}10^{-19}$ C. Hence 4,600 electrons will have:

$-1.602\text{x}10^{-19}\text{x}4600 = -7.369\text{x}10^{-16}$ C

Example 1.3
The total charge entering a terminal is given by q=5$t$sin4$\pi t mC$. Calculate the current at t=0.5 s.

Solution:

$$i = \frac{dq}{dt} = \frac{d}{dt}(5tsin4\pi t) = (5sin4\pi t + 20\pi tcos4\pi t)\ mA$$

At t=0.5 s.

$i = 31.42\ mA$

Example 1.4
Determine the total charge entering a terminal between t=1 s and t=2 s if the current passing the terminal is i= $(3t^2-t)\ A$.
Solution:

$$q = \int_{t=1}^{t=2} idt = \int_{1}^{2}(3t^2 - t)dt = \left(t^3 - \frac{t^2}{2}\right)_1^2 = (8-2) - \left(1 - \frac{1}{2}\right) = 5.5C$$

1.3.3 VOLTAGE (or) POTENTIAL DIFFERENCE
To move the electron in a conductor in a particular direction requires some work or energy transfer. This work is performed by an external electromotive force (emf), typically represented by the battery in Fig. 1.3. This emf is also known as voltage or potential difference. The voltage $_{ab}$between two points aand b in an electric circuit is the energy (or work) needed to move a unit charge from a to b.

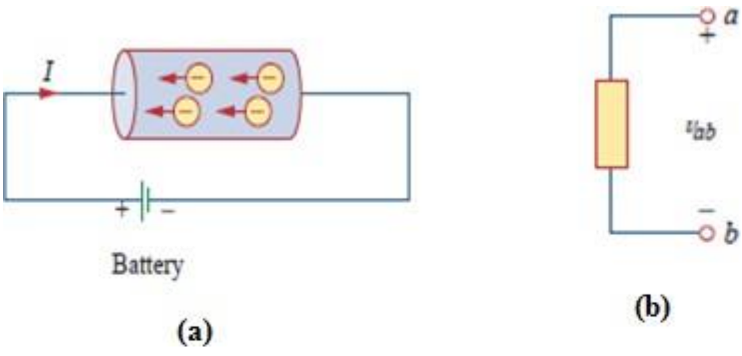


Fig. 1.3(a) Electric Current in a conductor, (b)Polarity of voltage $_{ab}$

Voltage (or potential difference) is the energy required to move charge from one point to the other, measured in volts (V). Voltage is denoted by the letter v or V.
Mathematically,

$$v_{ab} = \frac{dw}{dt}$$

where w is energy in joules (J) and q is charge in coulombs (C). The voltage $_{ab}$ or simply V is measured in volts (V).

1 volt = 1 joule/coulomb = 1 newton-meter/coulomb

Fig. 1.3 shows the voltage across an element (represented by a rectangular block) connected to points a and b. The plus (+) and minus (-) signs are used to define reference direction or voltage polarity. The $_{ab}$ can be interpreted in two ways: (1) point a is at a potential of $_{ab}$ volts higher than point b, or (2) the potential at point a with respect to point b is $_{ab}$. It follows logically that in general

$$v_{ab} = -v_{ba}$$

Voltage is always measured across a circuit element as shown in Fig. 1.4

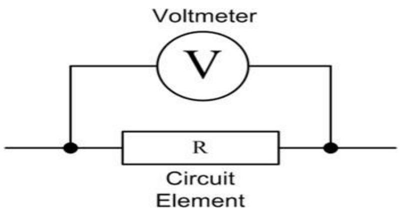


Fig. 1.4 Voltage across Resistor (R)

Example 1.5
An energy source forces a constant current of 2 A for 10 s to flow through a lightbulb. If 2.3 kJ is given off in the form of light and heat energy, calculate the voltage drop across the bulb.

Solution:
Total charge dq= i*dt = 2*10 = 20

The voltage drop is

$$v = \frac{dW}{dq} = \frac{2.3*10^3}{20} = 115\ V$$

1.3. 4 POWER

Power is the time rate of expending or absorbing energy, measured in watts (W). Power, is denoted by the letter p or P.
Mathematically,

$$p = \frac{dw}{dt}$$

Where p is power in watts (W), w is energy in joules (J), and t is time in seconds (s).

From voltage and current equations, it follows that;

$$p = \frac{dw}{dt} = \frac{dw}{dq} * \frac{dq}{dt} = V * I$$

Thus, if the magnitude of current I and voltage are given, then power can be evaluated as the product of the two quantities and is measured in watts (W).

Sign of power:

Plus sign: Power is absorbed by the element. (Resistor, Inductor)

Minus sign: Power is supplied by the element. (Battery, Generator)

Passive sign convention:

If the current enters through the positive polarity of the voltage, p = +vi
If the current enters through the negative polarity of the voltage, p = – vi

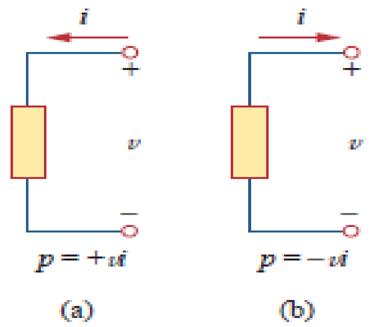


Fig 1.5 Polarities for Power using passive sign convention
(a) Absorbing Power (b) Supplying Power

1.3.5 ENERGY
Energy is the capacity to do work, and is measured in joules (J).
The energy absorbed or supplied by an element from time 0 to t is given by,

$$w = \int_0^t p dt = \int_0^t vi dt$$

The electric power utility companies measure energy in watt-hours (WH) or Kilo watt-hours (KWH)

1 WH = 3600 J

Example 1.6
A source e.m.f. of 5 V supplies a current of 3A for 10 minutes. How much energy is provided in this time?
Solution:

$$W = VIt = 5 \times 3 \times 10 \times 60 = 9\ kJ$$

Example 1.7
An electric heater consumes 1.8Mj when connected to a 250 V supply for 30 minutes. Find the power rating of the heater and the current taken from the supply.
Solution:
P=V /I = $(1.8\times10^6)/(30\times60)$ = 1000
Power rating of heater = 1kW
P =VI
Thus
I = P/V =1000/250=4
Hence the current taken from the supply is 4A.

1.4 OHM'S LAW

Georg Simon Ohm (1787–1854), a German physicist, is credited with finding the relationship between current and voltage for a resistor. This relationship is known as Ohm's law.

Ohm's law states that at constant temperature, the voltage (V) across a conducting material is directly proportional to the current (I) flowing through the material.
Mathematically,

$$V \alpha I$$

$$V=RI$$

Wherethe constant of proportionality R is called the resistance of the material. The V-I relation for resistor according to Ohm's law is depicted in Fig.1.6

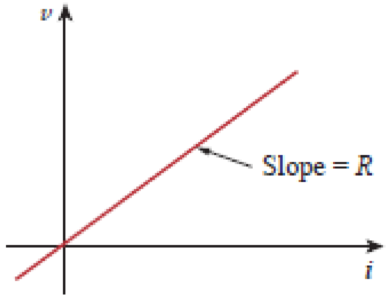


Fig. 1.6 V-I Characteristics for resistor

Limitations of Ohm's Law:

1. Ohm's law is not applicable to non-linear elements like diode, transistor etc.
2. Ohm's law is not applicable for non-metallic conductors like silicon carbide.

1.5 CIRCUIT ELEMENTS

An element is the basic building block of a circuit. An electric circuit is simply an interconnection of the elements. Circuit analysis is the process of determining voltages across (or the currents through) the elements of the circuit.

There are 2 types of elements found in electrical circuits.

a) Active elements (Energy sources): The elements which are capable of generating or delivering the energy are called active elements.
E.g., Generators, Batteries

b) Passive element (Loads): The elements which are capable of receiving the energy are called passive elements.
E.g., Resistors, Capacitors and Inductors

1.5.1 ACTIVE ELEMENTS (ENERGY SOURCES)

The energy sources which are having the capacity of generating the energy are called active elements. The most important active elements are voltage or current sources that generally deliver power/energy to the circuit connected to them.

There are two kinds of sources

a) Independent sources
b) Dependent sources

1.5.1.1 INDEPENDENT SOURCES:

An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit elements.

Ideal Independent Voltage Source:

An ideal independent voltage source is an active element that gives a constant voltage across its terminals irrespective of the current drawn through its terminals. In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. The symbol of idea independent voltage source and its V-I characteristics are shown in Fig. 1.7

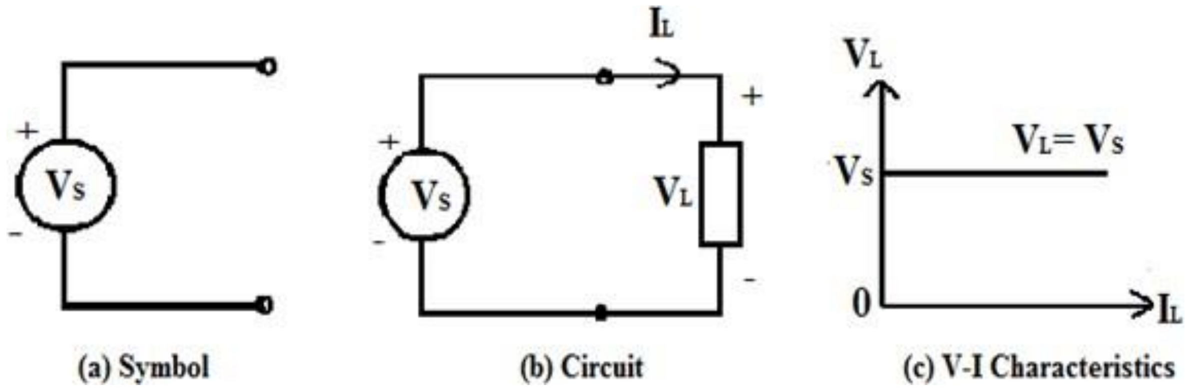


Fig. 1.7 Ideal Independent Voltage Source

Practical Independent Voltage Source:
Practically, every voltage source has some series resistance across its terminals known as internal resistance, and is represented by Rse. For ideal voltage source Rse = 0. But in practical voltage source Rse is not zero but may have small value. Because of this Rse voltage across the terminals decreases with increase in current as shown in Fig. 1.8

Terminal voltage of practical voltage source is given by

$$V_L = V_S - I_L R_{se}$$

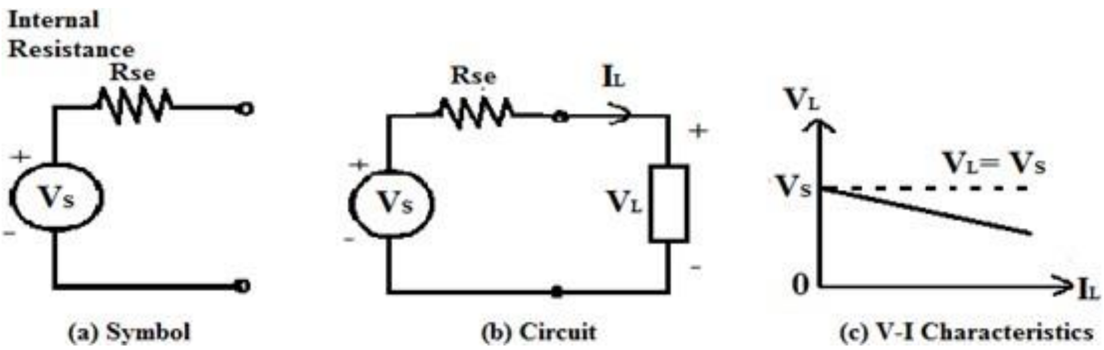


Fig. 1.8 Practical Independent Voltage Source

Ideal Independent Current Source:
An ideal independent Current source is an active element that gives a constant current through its terminals irrespective of the voltage appearing across its terminals. That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol of idea independent current source and its V-I characteristics are shown in Fig. 1.9

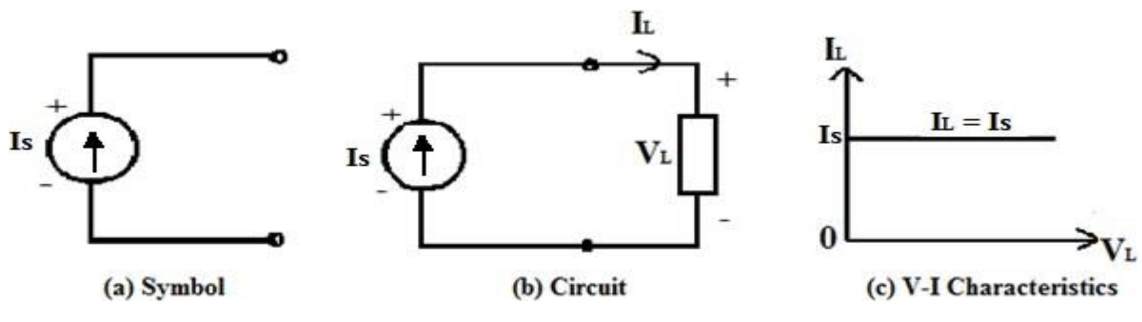


Fig. 1.9 Ideal Independent Current Source

Practical Independent Current Source:
Practically, every current source has some parallel/shunt resistance across its terminals known as internal resistance, and is represented by Rsh. For ideal current source Rsh = ∞ (infinity). But in practical voltage source Rsh is not infinity but may have a large value. Because of this Rsh current through the terminals slightly decreases with increase in voltage across its terminals as shown in Fig. 1.10.

Terminal current of practical current source is given by

$$I_L = Is - Ish$$

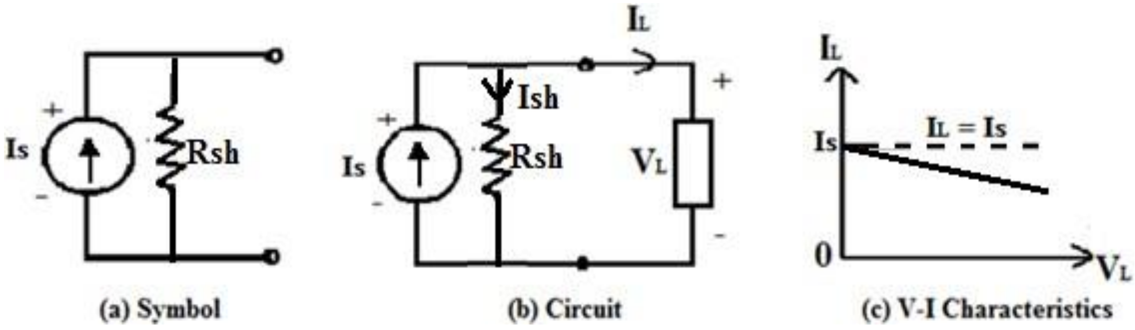


Fig. 1.10 Practical Independent Current Source

1.5.1.2 DEPENDENT (CONTROLLED) SOURCES

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1.11. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled voltage source (VCVS)
2. A current-controlled voltage source (CCVS)
3. A voltage-controlled current source (VCCS)
4. A current-controlled current source (CCCS)

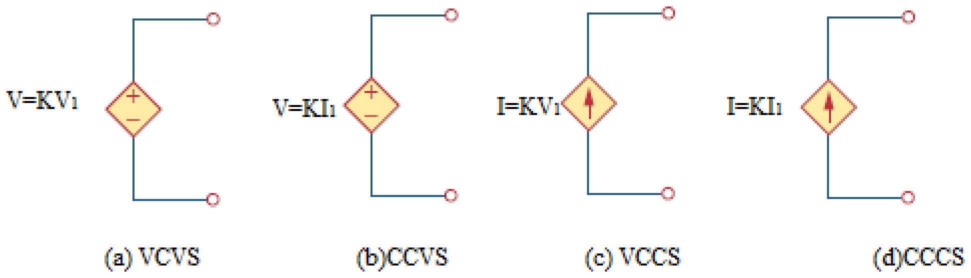


Fig. 1.11 Symbols for Dependent voltage source and Dependent current source

Dependent sources are useful in modeling elements such as transistors, operational amplifiers, and integrated circuits. An example of a current-controlled voltage source is shown on the right-hand side of Fig. 1.12, where the voltage 10i of the voltage source depends on the current i through element C. Students might be surprised that the value of the dependent voltage source is 10i V (and not 10i A) because it is a voltage source. The key idea to keep in mind is that a voltage source comes with polarities (+ -) in its symbol, while a current source comes with an arrow, irrespective of what it depends on.

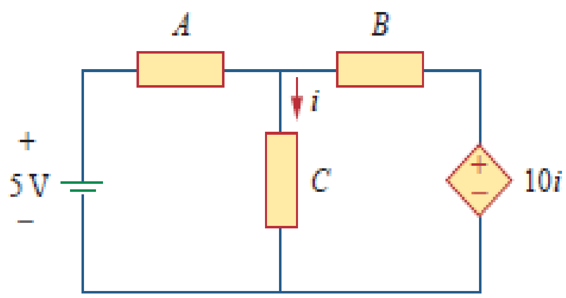


Fig. 1.12 The source in right hand side is current-controlled voltage source

1.5.2 PASSIVE ELEMENTS (LOADS)

Passive elements are those elements which are capable of receiving the energy. Some passive elements like inductors and capacitors are capable of storing a finite amount of energy, and return it later to an external element. More specifically, a passive element is defined as one that cannot supply average power that is greater than zero over an infinite time interval. Resistors, capacitors, Inductors fall in this category.

1.5.2.1RESISTOR

Materials in general have a characteristic behavior of resisting the flow of electric charge. This physical property, or ability to resist the flow of current, is known as resistance and is represented by the symbol R.The Resistance is measured in ohms (Ω). The circuit element used to model the current-resisting behavior of a material is called the resistor.

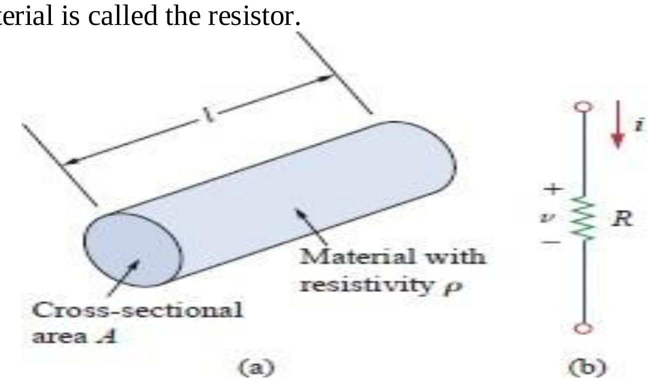


Fig. 1.13 (a) Typical Resistor, (b) Circuit Symbol for Resistor

The resistance of a resistor depends on the material of which the conductor is made and geometrical shape of the conductor. The resistance of a conductor is proportional to the its length (and inversely proportional to its cross sectional area (A). Therefore the resistance of a conductor can be written as,

$$R = \frac{\rho l}{A}$$

The proportionality constant is called the specific resistance o resistivity of the conductor and its value depends on the material of which the conductor is made.

The inverse of the resistance is called the conductance and inverse of resistivity is called specific conductance or conductivity. The symbol used to represent the conductance is G and conductivity is . Thus conductivity and its units are Siemens per meter

$$G = \frac{1}{R} = \frac{A}{\rho l} = \frac{1}{\rho}.\frac{A}{l} = \sigma.\frac{A}{l}$$

By using Ohm's Law, The power dissipated in a resistor can be expressed in terms of R as below

$$P = VI = I^2 R = \frac{V^2}{R}$$

The power dissipated by a resistor may also be expressed in terms of G as

$$P = VI = V^2 G = \frac{I^2}{G}$$

The energy lost in the resistor from time 0 to t is expressed as

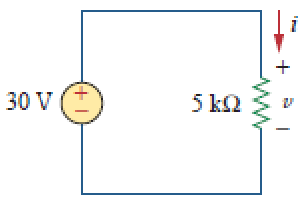
$$W = \int_0^t P dt$$

$$W = \int_0^t I^2 R dt = I^2 R t$$

$$W = \int_0^t \frac{V^2}{R} dt = \frac{V^2}{R} t$$

Where V is in volts, I is in amperes, R is in ohms, and energy W is in joules

Example 1.9
In the circuit shown in Fig. below, calculate the current i, the conductance G, the power p and energy lost in the resistor W in 2hours.



Solution:
The voltage across the resistor is the same as the source voltage (30 V) because the resistor and the voltage source are connected to the same pair of terminals. Hence, the current is

$$i = \frac{v}{R} = \frac{30}{5x10^3} = 6\ mA$$

The conductance is

$$G = \frac{1}{R} = \frac{1}{5x10^3} = 0.2mS$$

We can calculate the power in various ways

$$p = vi = 30(6x10^{-3}) = 180mW$$

or

$$p = i^2R = (6x10^{-3})^2(5x10^3) = 180mW$$

or

$$p = \frac{v^2}{R} = \frac{30^2}{5x10^3} = 180mW$$

Energy lost in the resistor is

$$W = i^2Rt = (6x10^{-3})^2(5x10^3)(2) = 360\ mWhor = 360\ mJ$$

1.5.2.2 INDUCTOR

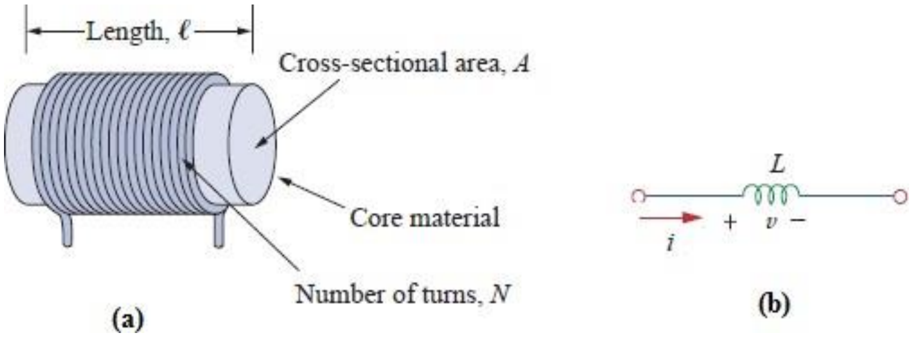


Fig. 1.14 (a) Typical Inductor, (b) Circuit symbol of Inductor

A wire of certain length, when twisted into a coil becomes a basic inductor. The symbol for inductor is shown in Fig.1.14 (b). If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field. Increase in current expands the fields, and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field, which induces a voltage across the coil according to Faraday's law of electromagnetic induction. i.e., the voltage across the inductor is directly proportional to the time rate of change of current.
Mathematically,

$$V \propto \frac{di}{dt}$$

$$v = L\frac{di}{dt}$$

Where L is the constant of proportionality called the inductance of an inductor. The unit of inductance is Henry (H).we can rewrite the above equation as

$$di = \frac{1}{L} v dt$$

Integrating both sides from time 0 to t, we get

$$\int_0^t di = \frac{1}{L}\int_0^t v\, dt$$

$$i(t) - i(0) = \frac{1}{L}\int_0^t v\, dt$$

$$i(t) = \frac{1}{L}\int_0^t v\, dt + i(0)$$

From the above equation we note that the current in an inductor is dependent upon the integral of the voltage across its terminal and the initial current in the coil$i(0)$.

The power absorbed by the inductor is

$$P = vi = Li\frac{di}{dt}$$

The energy stored by the inductor is

$$W = \int_0^t P dt$$

$$= \int_0^t Li\frac{di}{dt} dt = \frac{Li^2}{2}$$

From the above discussion, we can conclude the following.

1. The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to DC.
2. A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly i.e., the inductor opposes the sudden changes in currents.
3. The inductor can store finite amount of energy. Even if the voltage across the inductor is zero
4. A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

1.5.2.2 CAPACITOR

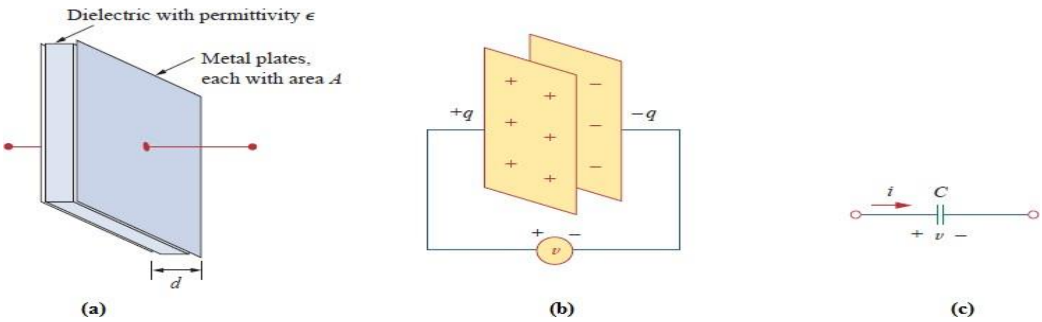


Fig. 1.15 (a) Typical Capacitor, (b) Capacitor connected to a voltage source, (c) Circuit Symbol of capacitor

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called electrodes, and the insulating medium is called dielectric. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges, and is concentrated within the dielectric.

When a voltage source v is connected to the capacitor, as in Fig 1.15 (c), the source deposits a positive charge q on one plate and a negative charge — q on the other. The capacitor is said to store the electric charge. The amount of charge stored, represented by q, is directly pro-proportional to the applied voltage v so that

Where C, the constant of proportionality, is known as the capacitance of the capacitor. The unit of capacitance is the farad (F).

Although the capacitance C of a capacitor is the ratio of the charge q per plate to the applied voltage v, it does not depend on q or v. It depends on the physical dimensions of the capacitor. For example, for the parallel-plate capacitor shown in Fig.1.15 (a), the capacitance is given by

$$q = Cv$$

Where A is the surface area of each plate, d is the distance between the plates, and is the permittivity of the dielectric material between the plates.
The current flowing through the capacitor is given by

$$C = \frac{\epsilon A}{d}$$

$$i = \frac{dq}{dt}$$

$$i = C\frac{dv}{dt}$$

We can rewrite the above equation as

$$dv = \frac{1}{C} i dt$$

Integrating both sides from time 0 to t, we get

$$\int_0^t dv = \frac{1}{C}\int_0^t i\, dt$$

$$v(t) - v(0) = \frac{1}{C}\int_0^t i\, dt$$

$$v(t) = \frac{1}{C}\int_0^t i\, dt + v(0)$$

From the above equation we note that the voltage across the terminals of a capacitor is dependent upon the integral of the current through it and the initial voltage.

The power absorbed by the capacitor is

$$P = vi = vC\frac{dv}{dt}$$

The energy stored by the capacitor is

$$W = \int_0^t P dt$$

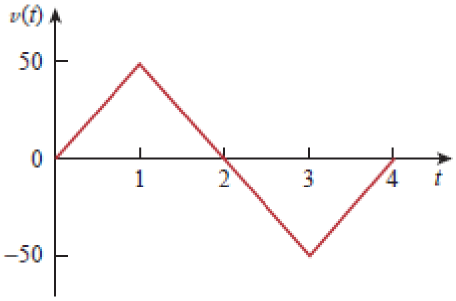
$$= \int_0^t vC\frac{dv}{dt}dt = \frac{Cv^2}{2}$$

From the above discussion we can conclude the following,

1. The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to DC.
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly. i.e., A capacitor will oppose the sudden changes in voltages.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy, but only stores it; that is why it is called non-dissipative passive element. However, physical capacitors dissipate power due to internal resistance.

Example 1.10

Determine the current through a 200 capacitor whose voltage is shown in Fig. below



Solution:

The voltage waveform can be described mathematically as

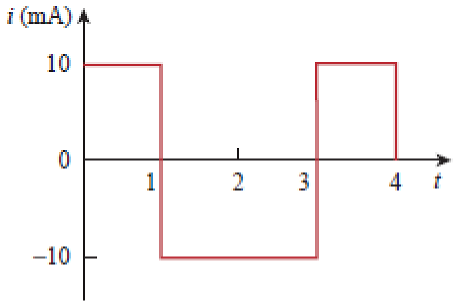
$$v(t) = \begin{cases} 50tV & 0 < t < 1 \\ 100 - 50tV & 1 < t < 3 \\ -200 + 50tV & 3 < t < 4 \\ 0 & otherwise \end{cases}$$

Since $i = C\frac{dv}{dt} and C = 200\mu F,$ we take the derivative of $v(t)$ to obtain the i(t)

$$i(t) = 200\text{x}10^{-6}\text{x}\begin{cases} 50 & 0 < t < 1 \\ -50 & 1 < t < 3 \\ 50 & 3 < t < 4 \\ 0 & otherwise \end{cases}$$

$$= \begin{cases} 10\ mA & 0 < t < 1 \\ -10\ mA & 1 < t < 3 \\ 10\ mA & 3 < t < 4 \\ 0 & otherwise \end{cases}$$

Hence, the current wave form is as shown in the fig. below



1.6 NETWORK/CIRCUIT TERMINOLOGY

In the following section various definitions and terminologies frequently used in electrical circuit analysis are outlined.

- **Network Elements:** The individual components such as a resistor, inductor, capacitor, diode, voltage source, current source etc. that are used in circuit are known as network elements.
- **Network:** The interconnection of network elements is called a network**.**
- **Circuit:** A network with at least one closed path is called a circuit. So, all the circuits are networks but all networks are not circuits.
- **Branch:** A branch is an element of a network having only two terminals.
- **Node:** A node is the point of connection between two or more branches. It is usually indicated by a dot in a circuit**.**
- **Loop:** A loop is any closed path in a circuit. A loop is a closed path formed by starting at a node, passing through a set of nodes, and returning to the starting node without passing through any node more than once.
- **Mesh or Independent Loop:** Mesh is a loop which does not contain any other loops in it.

1.7 KIRCHHOFF'S LAWS

The most common and useful set of laws for solving electric circuits are the Kirchhoff's voltage and current laws. Several other useful relationships can be derived based on these laws. These laws are formally known as Kirchhoff's current law (KCL) and Kirchhoff's voltage law (KVL).

1.7.1 KIRCHHOFF'S CURRENT LAW (KCL)

This is also called as Kirchhoff's first law or Kirchhoff's nodal law. Kirchhoff's first law is based on the law of conservation of charge, which requires that the algebraic sum of charges within a system cannot change.

Statement: Algebraic sum of the currents meeting at any junction or node is zero. The term 'algebraic' means the value of the quantity along with its sign, positive or negative.

Mathematically, KCL implies that

$$\sum_{n=1}^{N} i_n = 0$$

Where N is the number of branches connected to the node and is the nth current entering (or leaving) the node. By this law, currents entering a node may be regarded as positive, while currents leaving the node may be taken as negative or vice versa.

Alternate Statement: Sum of the currents flowing towards a junction is equal to the sum of the currents flowing away from the junction.

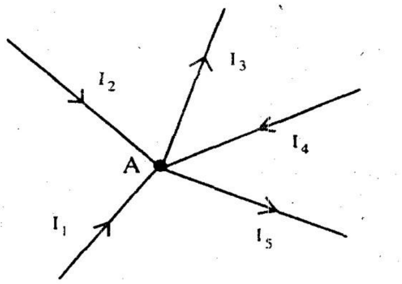


Fig 1.16 Currents meeting in a junction

Consider Fig. 1.16 where five branches of a circuit are connected together at the junction or node A. Currents I_1, I_2 and I_4 are flowing towards the junction whereas currents I_3 and I_5 are flowing away from junction A. If a positive sign is assigned to the currents I_2 and I_4 that are flowing into the junction then the currents I_3 and I_4 flowing away from the junction should be assigned with the opposite sign i.e. the negative sign.

Applying Kirchhoff's current law to the junction A

$$I_1 + I_2 - I_3 + I_4 - I_5 = 0 \text{ (algebraic sum is zero)}$$

The above equation can be modified as $I_1 + I_2 + I_4 = I_3 + I_5$ (sum of currents towards the junction = sum of currents flowing away from the junction).

1.7.2 KIRCHHOFF'S VOLTAGE LAW (KVL)

This is also called as Kirchhoff's second law or Kirchhoff's loop or mesh law. Kirchhoff's second law is based on the principle of conservation of energy.

Statement: Algebraic sum of all the voltages around a closed path or closed loop at any instant is zero. Algebraic sum of the voltages means the magnitude and direction of the voltages; care should be taken in assigning proper signs or polarities for voltages in different sections of the circuit.

Mathematically, KVL implies that

$$\sum_{n=1}^{N} V_n = 0$$

Where N is the number of voltages in the loop (or the number of branches in the loop) and is the n[th] voltage in a loop.

The polarity of the voltages across active elements is fixed on its terminals. The polarity of the voltage drop across the passive elements (Resistance in DC circuits) should be assigned with reference to the direction of the current through the elements with the concept that the current flows from a higher potential to lower potential. Hence, the entry point of the current through the passive elements should be marked as the positive polarity of voltage drop across the element and the exit point of the current as the negative polarity. The direction of currents in different branches of the circuits is initially marked either with the known direction or assumed direction.

After assigning the polarities for the voltage drops across the different passive elements, algebraic sum is accounted around a closed loop, either clockwise or anticlockwise, by assigning a particular sign, say the positive sign for all rising potentials along the path of tracing and the negative sign for all decreasing potentials. For example consider the circuit shown in Fig. 1.17

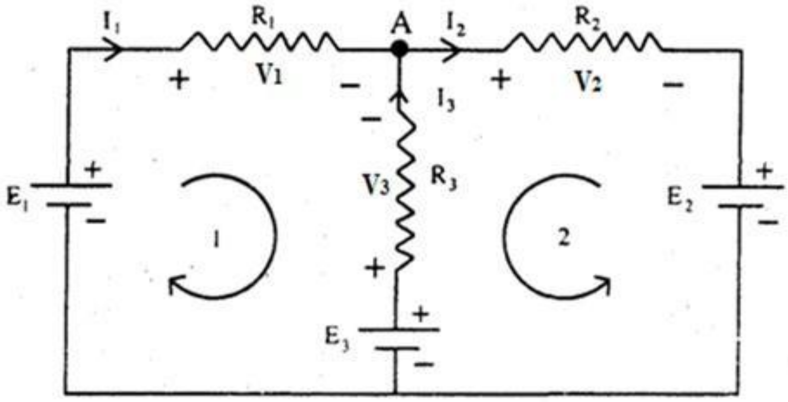


Fig. 1.17 Circuit for KVL

The circuit has three active elements with voltages E_1, E_2 and E_3. The polarity of each of them is fixed. R_1, R_2, R_3 are three passive elements present in the circuit. Currents I_1 and I_3 are marked flowing into the junction A and current I_2 marked away from the junction A with known information or assumed directions. With reference to the direction of these currents, the polarity of voltage drops V_1, V_2 and V_3 are marked.

For loop1 it is considered around clockwise

$$+ E_1 - V_1 + V_3 - E_3 = 0$$
$$+ E_1 - I_1 R_1 + I_3 R_3 - E_3 = 0$$
$$E_1 - E_3 = I_1 R_1 - I_3 R_3$$

For loop2 it is considered anticlockwise

$$+ E_2 + V_2 + V_3 – E_3 = 0$$
$$+ E_2 + I_2 R_2 + I_3 R_3 – E_3 = 0$$
$$E_2 – E_3 = - I_2 R_2 - I_3 R_3$$

Two equations are obtained following Kirchhoff's voltage law. The third equation can be written based on Kirchhoff's current law as

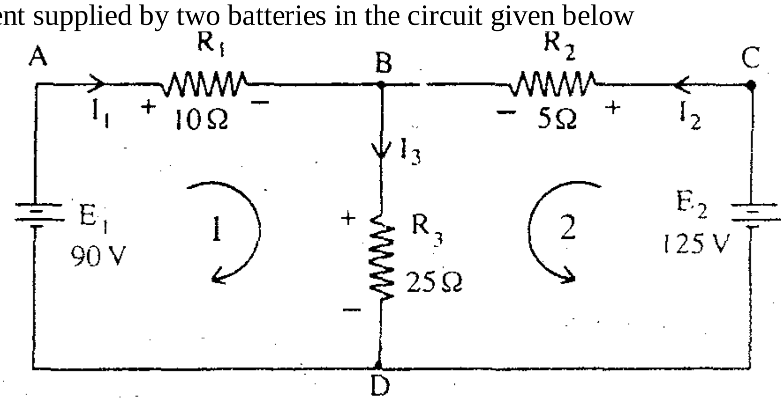
$$I_1 – I_2 + I_3 = 0$$

With the three equations, one can solve for the three currents I_1, I_2, and I_3.

If the results obtained for I_1, I_2, and I_3 are all positive, then the assumed direction of the currents are said to be along the actual directions. A negative result for one or more currents will indicate that the assumed direction of the respective current is opposite to the actual direction.

Example 1.11

Calculate the current supplied by two batteries in the circuit given below



Solution:

The four junctions are marked as A, B, C and D. The current through R_1 is assumed to flow from A to B and through R_2, from C to B and finally through R_3 from B to D. With reference to current directions, polarities of the voltage drop in R_1, R_2 and R_3 are then marked as shown in the figure. Applying KCL to junction B

$$I_3 = I_1 + I_2 \qquad \text{.....(1)}$$

Applying KVL to loop 1

$$E_1 – I_1R_1 – I_3R_3 = 0$$
$$I_1R_1 + I_3R_3 = E_1$$
$$10I_1 + 25I_3 = 90 \qquad \text{...... (2)}$$

Substituting Eq. (1) in Eq. (2)

$$10I_1 + 25(I_1 + I_2) = 90$$
$$35I_1 + 25I_2 = 90 \qquad \text{....... (3)}$$

Applying KVL to loop 2

$$E_2 – I_2R_2 – I_3R_3 = 0$$
$$I_2R_2 + I_3R_3 = E_2$$
$$5I_2 + 25I_3 = 125 \qquad \text{...... (4)}$$

Substituting Eq. (1) in Eq. (4)

$$5I_2 + 25(I_1 + I_2) = 125$$
$$25I_1 + 30I_2 = 125 \qquad \text{....... (5)}$$

Multiplying Eq. (3) by 6/5 we get

$$42I_1 + 30I_2 = 108 \qquad \text{...... (6)}$$

Subtracting Eq. (6) from Eq. (5)

$$-17I_1 = 17$$
$$I_1 = -1 \text{ A}$$

Substituting the value of I_1 in Eq. (5) we get

$$I_2 = 5 \text{ A}$$

As the sign of the current I_1 is found to be negative from the solution, the actual direction of I_1 is from B to A to D i.e. 90 V battery gets a charging current of 1 A.

1.8 RESISTIVE NETWORKS

1.8.1 SERIES RESISTORS AND VOLTAGE DIVISION

Two or more resistors are said to be in series if the same current flows through all of them. The process of combining the resistors is facilitated by combining two of them at a time. With this in mind, consider the single-loop circuit of Fig. 1.18.

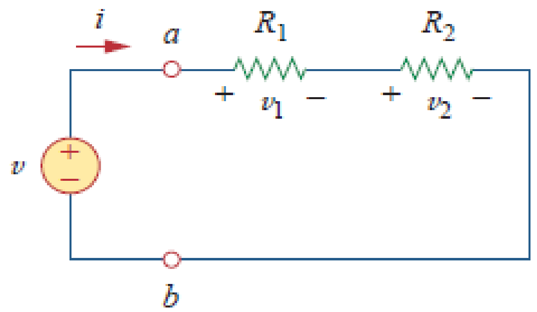


Fig.1.18 A single loop circuit with two resistors in series

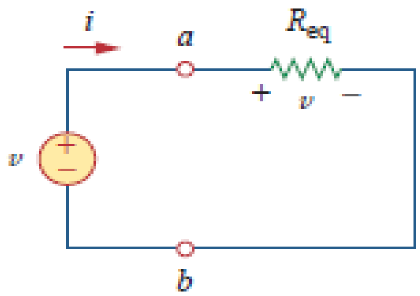


Fig. 1.19 Equivalent Circuit of series resistors

The two resistors are in series, since the same current i flow in both of them. Applying Ohm's law to each of the resistors, we obtain

$$v_1 = iR_1, v_2 = iR_2$$ ………. (1)

If we apply KVL to the loop (moving in the clockwise direction), we have

$$v - v_1 - v_2 = 0$$

……….. (2)

Combining equations (1) and (2), we get

……….. (3)

$$v = v_1 + v_2 = i(R_1 + R_2)$$

Or

$$i = \frac{v}{R_1 + R_2}$$ $$i = \frac{v}{R_1 + R_2}$$ ……….. (4)

Equation (3) can be written ………. (5)

as $v = iR_{eq}$

implying that the two resistors can be replaced by an equivalent resistor ;that is

$$R_{eq} = R_1 + R_2$$ …………… (6)

Thus, Fig. 1.18 can be replaced by the equivalent circuit in Fig. 1.19. The two circuits in Fig 1.18 and 1.19 are the equivalent because they because they exhibit the same voltage-current relationships at the terminals a-b. An equivalent circuit such as the one in Fig. 1.19 is useful in simplifying the analysis of a circuit.

In general, the equivalent resistance of any number of resistors connected in series is the sum of the individual resistances.

For N resistors in series then,

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N = \sum_{n=1}^{N} R_n$$

……… (7)

……… (7)

VOLTAGE DIVISION:

To determine the voltage across each resistor in Fig. 1.18, we substitute Eq. (4) into Eq. (1) and obtain

$$v_1 = \frac{v}{R_1 + R_2} R_1, v_2 = \frac{v}{R_1 + R_2} R_2$$

……… (8)

Notice that the source voltage v is divided among the resistors in direct proportion to their resistances; the larger the resistance, the larger the voltage drop. This is called the principle of voltage division, and the circuit in Fig. 1.18 is called a voltage divider. In general, if a voltage divider has N resistors ($R_1, R_2, \dots R_N$) in series with the source voltage v, the nth resistor (R_N) will have a voltage drop of

$$v_N = \frac{R_N}{R_1 + R_2 + \cdots + R_N}$$

……... (9)

1.8.2 PARALLEL RESISTORS AND CURRENT DIVISION

Two or more resistors are said to be in parallel if the same voltage appears across each element.Consider the circuit in Fig. 1.20, where two resistors are connected in parallel and therefore have the same voltage across them.

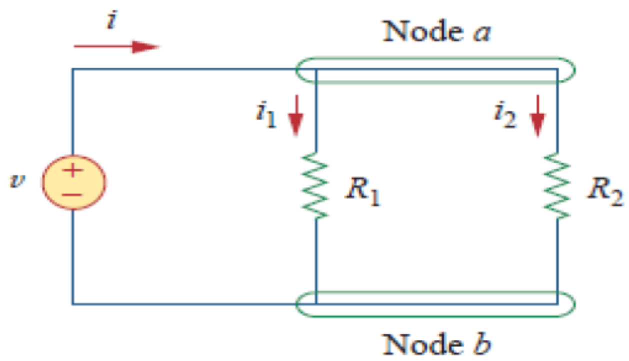


Fig. 1.20 Two resistors in parallel

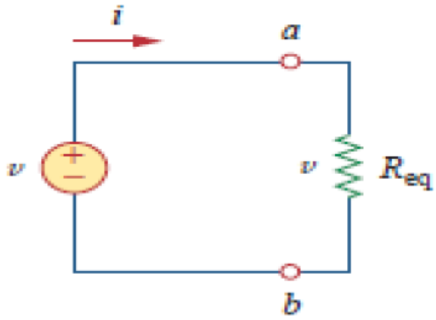


Fig. 1.21Equivalent circuit of Fig. 1.20

$$v = i_1R_1 = i_2R_2 \qquad \text{......... (1)}$$

$$i_1 = \frac{v}{R_1}, i_2 = \frac{v}{R_2} \qquad \text{.......... (2)}$$

Applying KCL at node a gives the total current i as

$$i = i_1 + i_2 \qquad \text{........ (3)}$$

Substituting Eq. (2) into Eq. (3), we get

$$i = \frac{v}{R_1} + \frac{v}{R_2} = v\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{v}{R_{eq}} \qquad \text{......... (4)}$$

where R_{eq} is the equivalent resistance of the resistors in parallel.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \text{.......... (5)}$$

Or

$$R_{eq} = \frac{R_1R_2}{R_1+R_2} \qquad \text{........... (6)}$$

Thus,
The equivalent resistance of two parallel resistors is equal to the product of their resistances divided by their sum.
It must be emphasized that this applies only to two resistors in parallel. From Eq. (6), if $R_1 = R_2, then R_{eq} = R_1/2$.
We can extend the result in Eq. (5) to the general case of a circuit with N resistors in parallel. The equivalent resistance is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{n=1}^{N} \frac{1}{R_n}$$

……… (7)

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}}$$

Thus,
The equivalent Resistance of parallel-connected resistors is the reciprocal of the sum of the reciprocals of the individual resistances.

Note that R_{eq} is always smaller than the resistance of the smallest resistor in the parallel combination.

Current Division:
Given the total current i entering node a in Fig. 1.20, then how do we obtain currents $i_1 and i_2$? We know that the equivalent resistor has the same voltage, or

$$v = iR_{eq} = \frac{iR_1R_2}{R_1 + R_2}$$

……. (8)

Substitute (8) in (2), we get

$$i_1 = \frac{iR_2}{R_1 + R_2}$$

$$i_2 = \frac{iR_1}{R_1 + R_2}$$

……….. (9)

This shows that the total current i is shared by the resistors in inverse proportion to their resistances. This is known as the principle of current division, and the circuit in Fig.1.20 is known as a current divider. Notice that the larger current flows through the smaller resistance.

1.9 INDUCTIVE NETWORKS

Now that the inductor has been added to our list of passive elements, it is Necessary to extend the powerful tool of series-parallel combination. We need to know how to find the equivalent inductance of a series-connected or parallel-connected set of inductors found in practical circuits.

1.9.1 SERIES INDUCTORS

Two or more inductors are said to be in series, if the same current flows through all of them. Consider a series connection of N inductors, as shown in Fig. 1.22(a), with the equivalent circuit shown in Fig. 1.22(b). The inductors have the same current through them.

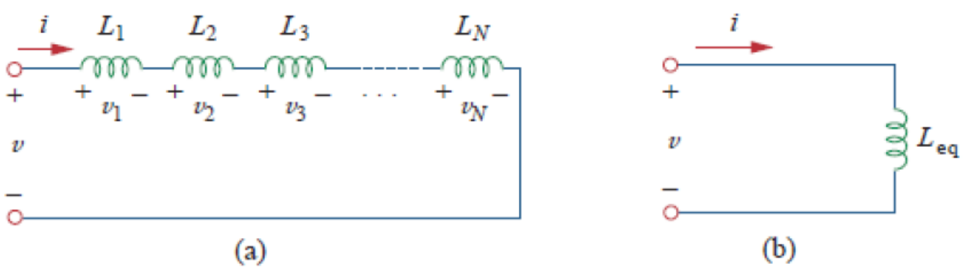


Fig. 1.22 (a) series connection of N inductors (b) Equivalent circuit for the series inductors

Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \cdots + v_N \text{ …… (1)}$$

We know that the voltage across an inductor is $v = L\frac{di}{dt}$

Therefore, Eq. (1) becomes

$$v = L_1\frac{di}{dt} + L_2\frac{di}{dt} + L_3\frac{di}{dt} + \cdots + L_N\frac{di}{dt}$$

$$= (L_1 + L_2 + L_3 + \cdots + L_N)\frac{di}{dt} \dots\dots\dots (2)$$

$$= \sum_{n=1}^{N}(L_n)\frac{di}{dt} = L_{eq}\frac{di}{dt}$$

Where,

$$L_{eq} = (L_1 + L_2 + L_3 + \cdots + L_N \dots\dots\dots (3)$$

Thus

The equivalent inductance of series-connected inductors is the sum of the individual inductances.

* Inductors in series are combined in exactly the same way as resistors in series.

1.9.2 INDUCTORS IN PARALLEL

Two or more inductors are said to be in parallel, if the same voltage appears across each element. We now consider a parallel connection of N inductors, as shown in Fig. 1.23(a), with the equivalent circuit in Fig. 1.23(b). The inductors have the same voltage across them.

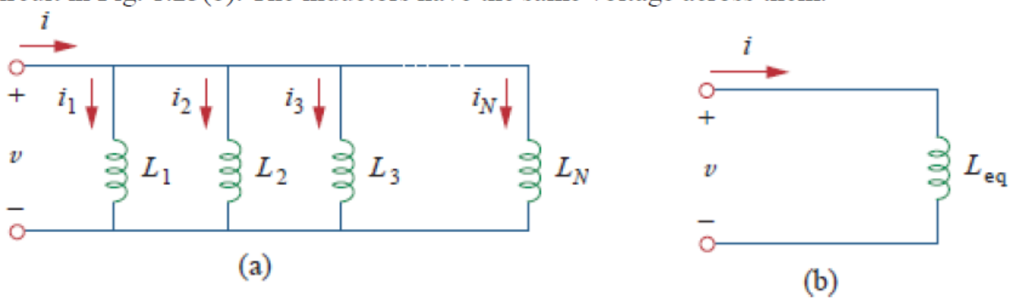


Fig. 1.23 (a) Parallel connection of N inductors (b) Equivalent circuit for parallel inductors

Using KCL,

$$i = i_1 + i_2 + i_3 + \cdots + i_N \dots\dots\dots (1)$$

But the current through the inductor is

$$i = \frac{1}{L}\int_0^t v dt + i(0)$$

If we neglect the initial value of current i.e, $i(0) = 0$ then current through inductor becomes

$$i = \frac{1}{L}\int_0^t v dt$$

Hence,

$$i = \frac{1}{L_1}\int_0^t v dt + \frac{1}{L_2}\int_0^t v dt + \frac{1}{L_3}\int_0^t v dt + \cdots + \frac{1}{L_N}\int_0^t v dt$$

$$i = \left(\frac{1}{L_1} + \frac{1}{L_3} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}\right)\int_0^t vdt$$

$$\therefore i = \left(\sum_{n=1}^{N} \frac{1}{L_n}\right)\int_0^t vdt = \frac{1}{L_{eq}}\int_0^t vdt \dots\dots\dots (2)$$

Where,

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_3} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}$$

$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_3} + \frac{1}{L_3} + \cdots + \frac{1}{L_N}}$$

Thus,

The equivalent inductance of parallel inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

* Note that the inductors in parallel are combined in the same way as resistors in parallel.

1.10 CAPACITIVE NETWORKS

We know from resistive circuits and inductive circuits that the series-parallel combination is a powerful tool for reducing circuits. This technique can be extended to series-parallel connections of capacitors, which are sometimes encountered. We desire to replace these capacitors by a single equivalent capacitor $C_{eq.}$

1.10.1 SERIES CAPACITORS

Two or more capacitors are said to be in series, if the same current flows through all of them. Consider a series connection of N capacitors, as shown in Fig. 1.24(a), with the equivalent circuit shown in Fig. 1.24(b). The capacitors have the same current through them.

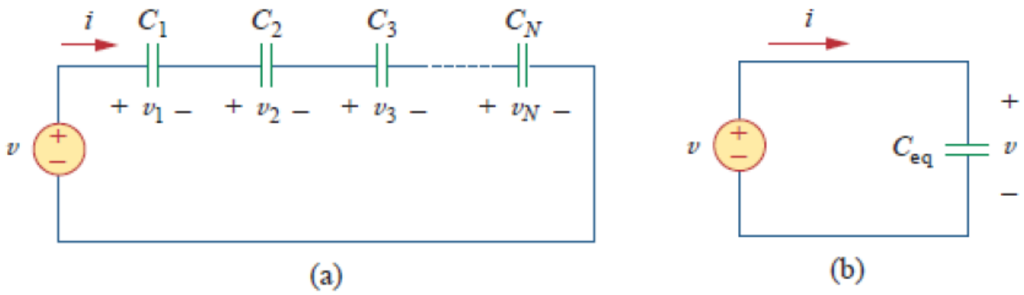


Fig. 1.24 (a) series connection of N capacitors (b) Equivalent circuit for the series capacitors

Applying KVL to the loop,

$$v = v_1 + v_2 + v_3 + \cdots + v_N \dots\dots (1)$$

We know that the voltage across a capacitor is

$$v = \frac{1}{C}\int_0^t idt + v(0)$$

If we neglect the initial value of voltage i.e, $v(0) = 0$ then voltage across the capacitor becomes

$$v = \frac{1}{C}\int_0^t idt$$

Hence, Eq. (1) becomes

$$v = \frac{1}{C_1}\int_0^t idt + \frac{1}{C_2}\int_0^t idt + \frac{1}{C_3}\int_0^t idt + \cdots + \frac{1}{C_N}\int_0^t idt$$

$$v = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}\right)\int_0^t idt$$

$$\therefore v = \left(\sum_{n=1}^{N}\frac{1}{C_n}\right)\int_0^t idt = \frac{1}{C_{eq}}\int_0^t idt \dots\dots\dots(2)$$

Where,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}$$

$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N}}$$

Thus,

The equivalent capacitance of series-connected capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.

* Note that the capacitors in series are combined in the same way as resistors in parallel.

For N=2,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

1.10.2 PARALLEL CAPACITORS

Two or more capacitors are said to be in parallel, if the same voltage appears across each element. Consider a parallel connection of N capacitors, as shown in Fig. 1.25(a), with the equivalent circuit in Fig. 1.25(b). The capacitors have the same voltage across them.

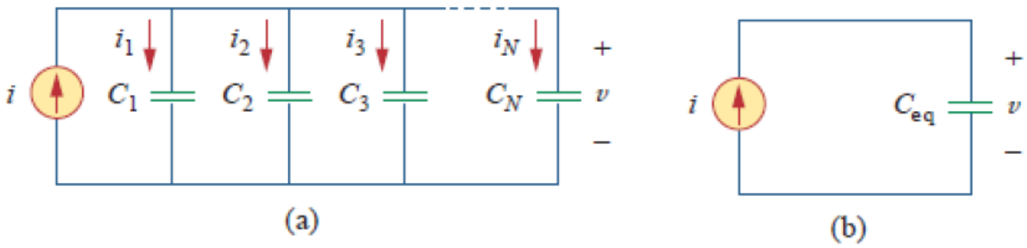


Fig. 1.25 (a) Parallel connection of N capacitors (b) Equivalent circuit for parallel capacitors

Applying KCL to Fig. 1.25(a)

$$i = i_1 + i_2 + i_3 + \cdots + i_N \dots\dots\dots(1)$$

We know that the current through capacitor is

$$\mathrm{i} = \mathrm{C}\frac{\mathrm{dv}}{\mathrm{dt}}$$

Therefore, Eq. (1) becomes

$$\mathrm{i} = \mathrm{C_1}\frac{\mathrm{dv}}{\mathrm{dt}} + \mathrm{C_2}\frac{\mathrm{dv}}{\mathrm{dt}} + \mathrm{C_3}\frac{\mathrm{dv}}{\mathrm{dt}} + \cdots + \mathrm{C_N}\frac{\mathrm{dv}}{\mathrm{dt}}$$

$$= (C_1 + C_2 + C_3 + \cdots + C_N)\frac{dv}{dt} \ldots\ldots\ldots (2)$$

$$= \sum_{n=1}^{N}(C_n)\frac{dv}{dt} = C_{eq}\frac{dv}{dt}$$

Where,

$$C_{eq} = (C_1 + C_2 + C_3 + \cdots + C_N) \qquad \ldots\ldots\ldots (3)$$

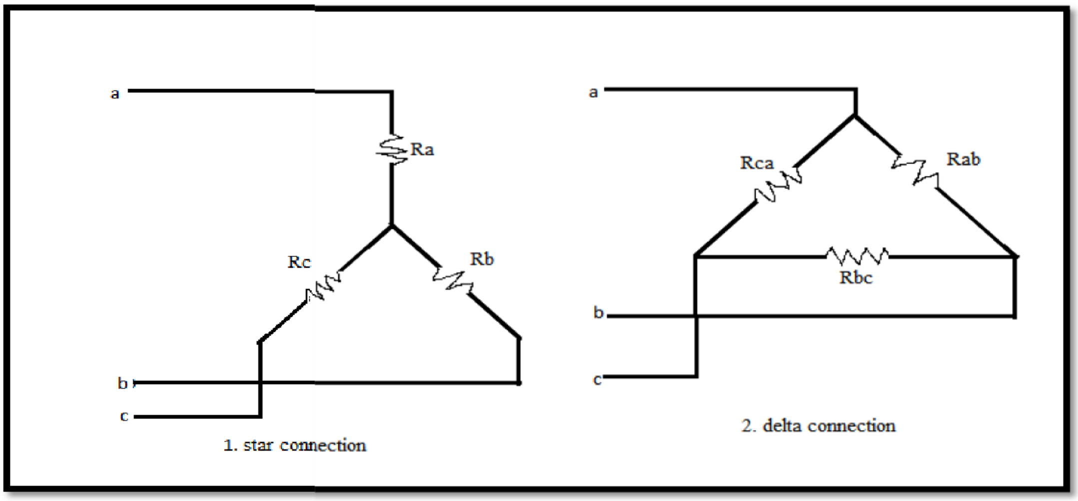
Thus

The equivalent capacitance of parallel-connected capacitors is the sum of the individual capacitances.

* Capacitors in parallel are combined in exactly the same way as resistors in series.

STAR – DELTA AND DELTA – STAR TRANSFORMATION

If there are three resistances are connected to a common point in the form as shown in fig (1). They are said to be star connected and if they are connected as shown in fig (2) they are said to be delta connected.



- In order to reduce the networks, it may be necessary to replace a star connected set of resistances by an equivalent delta connected set of resistances vice versa.

- The star delta transformation technique is useful in solving complex networks. Basically, any three circuit elements, i.e. Resistive, Inductive or capacitive, may be connected in two different ways. One way of connecting these elements is called the star connection, or the Y connection. The other way of connecting these elements is called delta connection or Δ connection.

The equivalence between the above two networks is obtained by equating the effective resistance between the corresponding terminals for the two networks.

Equating the resistances between corresponding pairs of terminals,Between a & b

Ra + Rb = Rab (Rbc +Rca) / (Rab +Rbc +Rca) ------------------------- (1)

Between b & c

$$R_b + R_c = R_{bc} (R_{ca} + R_{ab}) / (R_{ab} + R_{bc} + R_{ca}) \text{ ------------------------- } (2)$$

Between c & a

$$R_c + R_a = R_{ca} (R_{ab} + R_{bc}) / (R_{ab} + R_{bc} + R_{ca}) \text{ --------------------------} (3)$$

Subtracting (2) from (1) we get

$$R_a - R_c = R_{ca} (R_{ab} - R_{bc}) / (R_{ab} + R_{bc} + R_{ca}) \text{ --------------------------} (4)$$

Adding (3) and (4) we get

$$R_a = R_{ca} . R_{ab} / (R_{ab} + R_{bc} + R_{ca}) \text{ ----------------------- } (5)$$

Similarly,

$$R_b = R_{bc} . R_{ab} / (R_{ab} + R_{bc} + R_{ca}) \text{ -----------------------} (6)$$

$$R_c = R_{ca} . R_{bc} / (R_{ab} + R_{bc} + R_{ca}) \text{ ----------------------- } (7)$$

Equations (5), (6) & (7) to transform delta – star i.e. we can obtain an equivalent star connected resistances for the given delta connected resistances.

From the above equations, we get

$$R_aR_b + R_bR_c + R_cR_a = R_{ab} . R_{bc} . R_{ca} / (R_{ab} + R_{bc} + R_{ca}) \text{ ----------------------------} (8)$$

Dividing (8) by Ra i.e. equation (5)

$$R_aR_b + R_bR_c + R_cR_a = R_{bc}$$

.Ra Cancel Ra on both sides & we get

$$R_b + R_c + (R_b.R_c/R_a) = R_{bc} \text{ ------------------------ } (9)$$

Simillarly dividing equation (8) by equation Rb & Rc, we got

$$R_a + R_b + (R_a.R_b/R_c) = R_{ab} \text{ ------------------------ } (10)$$

$$R_c + R_a + (R_a.R_c/R_b) = R_{ca} \text{ ------------------------} (11)$$

From the above equations (9),(10) ,(11) we can replace a star connected resistances by an equivalent delta connected resistances.

5.2. Maxwell's Loop (or Mesh) Current Method

The method of *loop* or *mesh* currents is generally used in solving networks having some degree of complexity. Such a degree of complexity already begins for a network of three meshes. It might even be convenient at times to use the method of loop or mesh currents for solving a two-mesh circuit.

The *mesh-current method* is preferred to the general or branch-current method because the unknowns in the initial stage of solving a network are equal to the number of meshes, *i.e.*, the mesh currents. The necessity of writing the node-current equations, as done in the general or branch-current method where branch currents are used, is *obviated*. There are as many mesh-voltage equations as these are independent loop or mesh, currents. Hence, the M-mesh currents are obtained by solving the M-mesh voltages or loop equations for M unknowns. After solving for the mesh currents, only a matter of resolving the confluent mesh currents into the respective branch currents by very simple algebraic manipulations is required.

This method eliminates a great deal of tedious work involved in branch-current method and is best suited when energy sources are voltage sources rather than current sources. This method can be used only for *planar circuits*.

The **procedure** for writing the equations is as follows :

1. Assume the smallest number of mesh currents so that at least one mesh current links every element. As a matter of convenience, all mesh currents are assumed to have a *clockwise direction*.

The *number of mesh currents is equal to the number of meshes in the circuit*.

2. For each mesh write down the Kirchhoff's voltage law equation. Where more than one mesh current flows through an element, the algebraic sum of currents should be used. The algebraic sum of mesh currents may be sum or the difference of the currents flowing through the element depending on the direction of mesh currents.

3. Solve the above equations and from the mesh currents find the branch currents.

Fig. 46 shows two batteries E_1 and E_2 connected in a network consisting of three resistors. Let the loop currents for two meshes be I_1 and I_2 (both clockwise-assumed). It is obvious that current through R_3 (when considered as a part of first loop) is $(I_1 - I_2)$. However, when R_3 is considered part of the second loop, current through it is $(I_2 - I_1)$.

Applying Kirchhoff's voltage law to the *two loops*, we get

$$E_1 - I_1R_1 - R_3(I_1 - I_2) = 0$$

or $$E_1 - I_1(R_1 + R_3) + I_2R_3 = 0 \quad \textit{...Loop 1}$$

Similarly, $$-I_2R_2 - E_2 - R_3(I_2 - I_1) = 0$$

$$-I_2R_2 - E_2 - I_2R_3 + I_1R_3 = 0$$

or $$I_1R_3 - I_2(R_2 + R_3) - E_2 = 0 \quad \textit{...Loop 2}$$

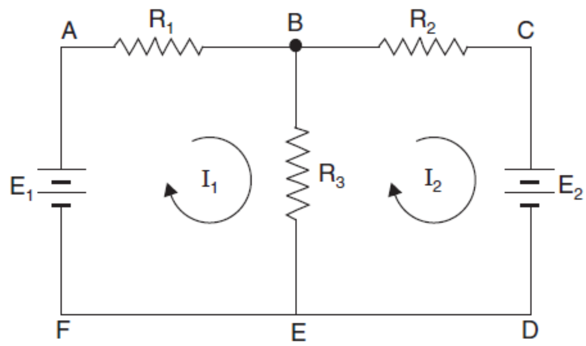


Fig. 46

The above two equations can be solved not only to find loop currents but branch currents as well.

Example 33. *Determine the currents through various resistors of the circuit shown in Fig. 47 using the concept of mesh currents.*

Solution. Refer Fig. 47.

Since there are two meshes, let the loop currents be as shown.

Applying Kirchhoff's law to *loop 1*, we get

$$24 - 4I_1 - 2(I_1 - I_2) = 0$$
$$-6I_1 + 2I_2 + 24 = 0$$

or
$$3I_1 - I_2 = 12 \quad ...(i)$$

For *loop 2*, we have

$$-2(I_2 - I_1) - 6I_2 - 12 = 0$$
$$2I_1 - 8I_2 - 12 = 0$$
$$I_1 - 4I_2 = 6 \quad ...(ii)$$

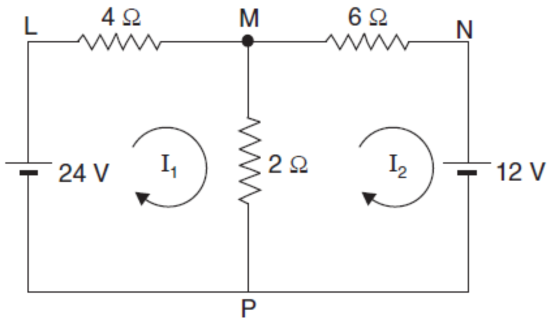


Fig. 47

Solving (*i*) and (*ii*), we get $I_1 = \frac{42}{11}$ A

and $I_2 = -\frac{6}{11}$ A

Hence Current through 4 Ω resistor = $\mathbf{\frac{42}{11}}$ **A (from L to M). (Ans.)**

Current through 6 Ω resistor = $\mathbf{\frac{6}{11}}$ **A (from N to M). (Ans.)**

Current through 2 Ω resistor = $\frac{42}{11} - \left(-\frac{6}{11}\right) = \mathbf{\frac{48}{11}}$ **A (from M to P). (Ans.)**

Example 34. *Determine the current supplied by each battery in the circuit shown in Fig. 48.*

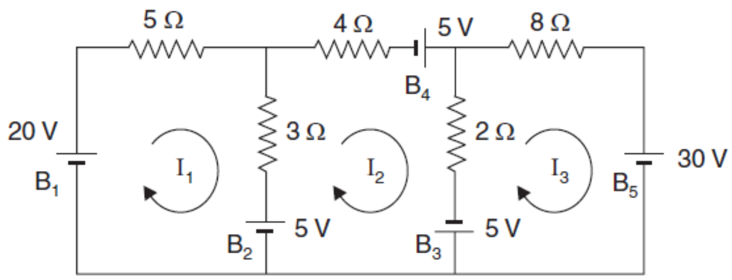


Fig. 48

Solution. Refer to Fig. 48.

As there are three meshes, let the three loop currents be as shown.

Applying Kirchhoff's law to *loop 1*, we get

$$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$$

or
$$8I_1 - 3I_2 = 15 \quad ...(i)$$

For *loop 2*, we have

$$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$$
$$3I_1 - 9I_2 + 2I_3 = -15 \quad ...(ii)$$

For *loop 3*, we have

$$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$$
$$2I_2 - 10I_3 = 35 \quad ...(iii)$$

Eliminating I_1 from (*i*) and (*ii*), we get

$$63I_2 - 16I_3 = 165 \quad ...(iv)$$

Solving (*iii*) and (*iv*), we get

$$I_2 = 1.82 \text{ A} \quad \text{and } I_3 = -3.15 \text{ A}$$

(– ve sign means direction of current is *counter-clockwise*)

Substituting the value of I_2 in (*i*), we get

$$I_1 = 2.56 \text{ A}$$

Current through battery B_1 (discharging current) $= I_1 =$ **2.56 A. (Ans.)**

Current through battery B_2 (charging current) $= I_1 - I_2 = 2.56 - 1.82 =$ **0.74 A. (Ans.)**

Current through battery B_3 (discharging current) $= I_2 + I_3 = 1.82 + 3.15 =$ **4.97 A. (Ans.)**

Current through battery B_4 (discharging current) $= I_2 =$ **1.82 A. (Ans.)**

Current through battery B_5 (discharging current) $= I_3 =$ **3.15 A. (Ans.)**

Example 35. *Determine the currents through the different branches of the bridge circuit shown in Fig. 49.*

Solution. Refer to Fig. 49.

The three mesh currents are assumed as shown.

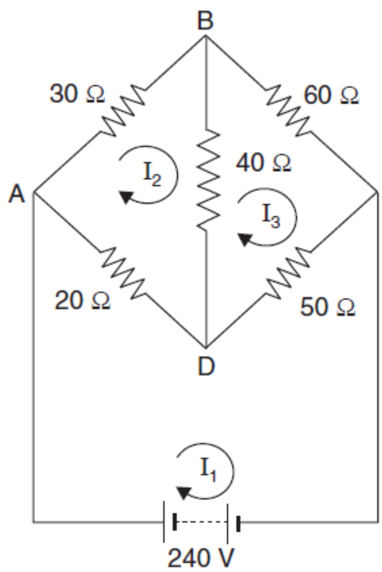


Fig. 49

The equations for the three meshes are :

For loop 1 : $240 - 20(I_1 - I_2) - 50(I_1 - I_3) = 0$

or $-70I_1 + 20I_2 + 50I_3 = -240$

or $70I_1 - 20I_2 - 50I_3 = 240 \quad ...(i)$

For loop 2 : $-30I_2 - 40(I_2 - I_3) - 20(I_2 - I_1) = 0$

or $20I_1 - 90I_2 + 40I_3 = 0$

or $2I_1 - 9I_2 + 4I_3 = 0 \quad ...(ii)$

For loop 3 : $-60I_3 - 50(I_3 - I_1) - 40(I_3 - I_2) = 0$

$50I_1 + 40I_2 - 150I_3 = 0$

$5I_1 + 4I_2 - 15I_3 = 0 \quad ...(iii)$

Solving these equations, we get

$$I_1 = 6.10 \text{ A}, I_2 = 2.56 \text{ A}, I_3 = 2.72 \text{ A}$$

Current through 30 Ω resistor $= I_2$

= **2.56 A (A to B). (Ans.)**

Current through 60 Ω resistor $= I_3 =$ **2.72 A (B to C). (Ans.)**

Current through 20 Ω resistor $= I_1 - I_2 = 6.10 - 2.56 =$ **3.54 A (A to D). (Ans.)**

Current through 50 Ω resistor $= I_1 - I_3 = 6.10 - 2.72 =$ **3.38 A (D to C). (Ans.)**

Current through 40 Ω resistor $= I_3 - I_2 = 2.72 - 2.56 =$ **0.16 A (D to B). (Ans.)**

5.3. Nodal Voltage Method

Under this method the following ***procedure*** is adopted :

1. Assume the voltages of the different independent nodes.
2. Write the equations for each mode as per Kirchhoff's current law.
3. Solve the above equations to get the node voltages.
4. Calculate the branch currents from the values of node voltages.

Let us consider the circuit shown in the Fig. 52. L and M are the two independent nodes ; M can be taken as the reference node. Let the voltage of node L (with respect to M) be V_L.

Using Kirchhoff's law, we get

$$I_1 + I_2 = I_3 \quad ...(22)$$

Ohm's law gives $I_1 = \frac{V_1}{R_1} = \frac{(E_1 - V_L)}{R_1}$

$$I_2 = \frac{V_2}{R_2} = \frac{(E_2 - V_L)}{R_2} \quad ...(23)$$

$$I_3 = \frac{V_3}{R_3} = \frac{V_L}{R_3}$$

$$\frac{E_1 - V_L}{R_1} + \frac{E_2 - V_L}{R_2} = \frac{V_L}{R_3} \quad ...(24)$$

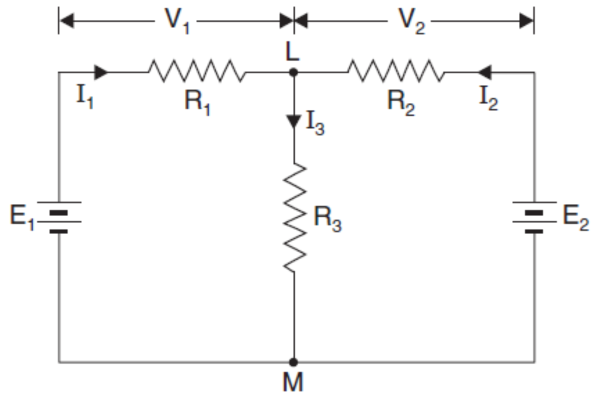


Fig. 52

Rearranging the terms, we get

$$V_L\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right] - \frac{E_1}{R_1} - \frac{E_2}{R_2} = 0 \quad ...(25)$$

It may be noted that the above nodal equation contains the following terms :

(*i*) The node voltage multiplied by the sum of all conductances connected to that anode. This term is *positive*.

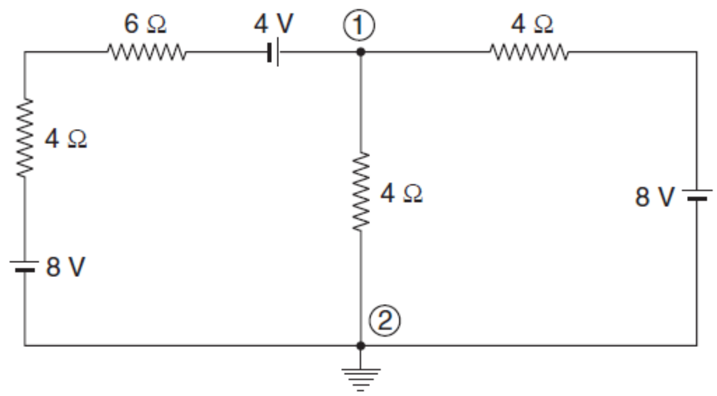
(*ii*) The node voltage at the other end of each branch (connected to this node) multiplied by the conductance of branch. These terms are *negative*.

— In this method of solving a network the *number of equations required for the solution is one less than the number of independent nodes in the network*.

— In general the nodal analysis *yields similar solutions*.

— The nodal method is very suitable for *computer work*.

Example 38. *Using Node voltage method, find the current in 6 Ω resistance for the network shown in Fig. 54.*



Solution. Refer Fig. 54. Considering node 2 as the reference node and using node voltage method, we have

$$V_1\left[\frac{1}{(6+4)} + \frac{1}{4} + \frac{1}{4}\right] - \frac{8}{4} - \left(\frac{8+4}{10}\right) = 0$$

(The reason for adding the two battery voltages of 4 V and 8 V is because they are connected in additive series).

or $\quad V_1\,(0.1 + 0.25 + 0.25) - 2 - 1.2 = 0$

$\therefore \quad V_1 = 5.33$ V

The current flowing through the 6 Ω resistance towards node 1 is

$$= \frac{12 - 5.33}{6 + 4} = \mathbf{0.667\ A.\ (Ans.)}$$

Example 37. *For the circuit shown in Fig. 53, find the currents through the resistances R_3 and R_4.*

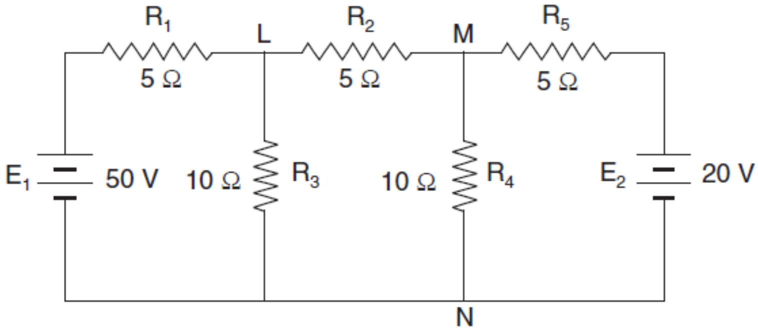


Fig. 53

Solution. Refer Fig. 53.

Let L, M and N = Independent nodes, and

V_L and V_M = Voltages of nodes L and M with respect to node N.

The nodal equations for the nodes L and M are :

$$V_L\left[\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right]-\frac{E_1}{R_1}-\frac{V_M}{R_2}=0 \quad ...(i)$$

$$V_M\left[\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right]-\frac{E_2}{R_5}-\frac{V_L}{R_2}=0 \quad ...(ii)$$

Substituting the values in (*i*) and (*ii*) and simplifying, we get

$$V_L\left(\frac{1}{5}+\frac{1}{5}+\frac{1}{10}\right)-\frac{50}{5}-\frac{V_M}{5}=0$$

or $$2.5V_L - V_M - 50 = 0 \quad ...(iii)$$

and $$V_M\left(\frac{1}{5}+\frac{1}{10}+\frac{1}{5}\right)-\frac{20}{5}-\frac{V_L}{5}=0$$

or $$2.5V_M - V_L - 20 = 0$$

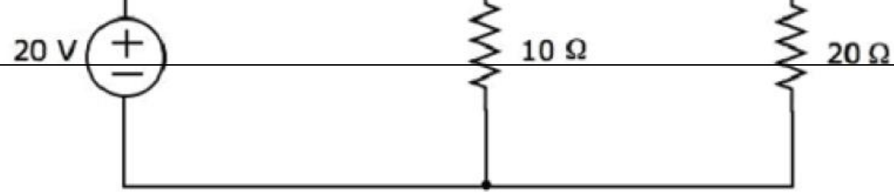
or $$-V_L + 2.5V_M - 20 = 0 \quad ...(iv)$$

Solving (*iii*) and (*iv*), we get

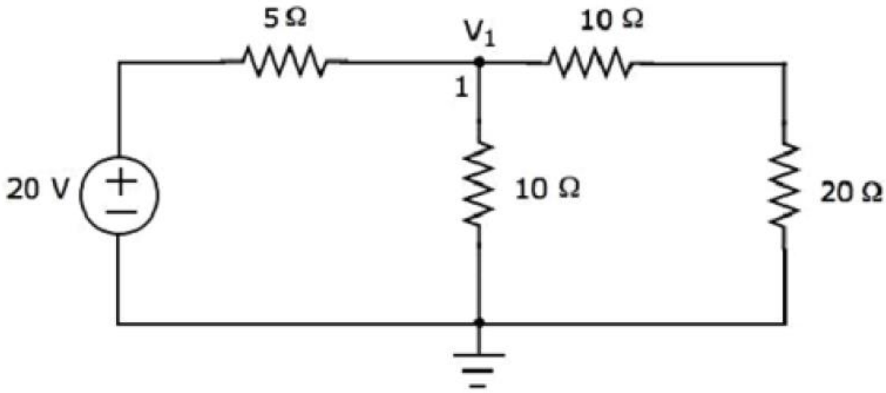
$$V_L = 27.6\ \text{V},\ V_M = 19.05\ \text{V}$$

Current through $\mathbf{R_3}$ $= \dfrac{V_L}{R_3} = \dfrac{27.8}{10} = \mathbf{2.76\ A.\ (Ans.)}$

Current through $\mathbf{R_4}$ $= \dfrac{V_M}{R_4} = \dfrac{19.05}{10} = \mathbf{1.905\ A.\ (Ans.)}$



There is only one principal node except Ground in the above circuit. So, we can use **nodal analysis**method. The node voltage V_1 is labelled in the following figure. Here, V_1 is the voltage from node 1 with respect to ground.



The **nodal equation** at node 1 is

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1}{10 + 20} = 0$$

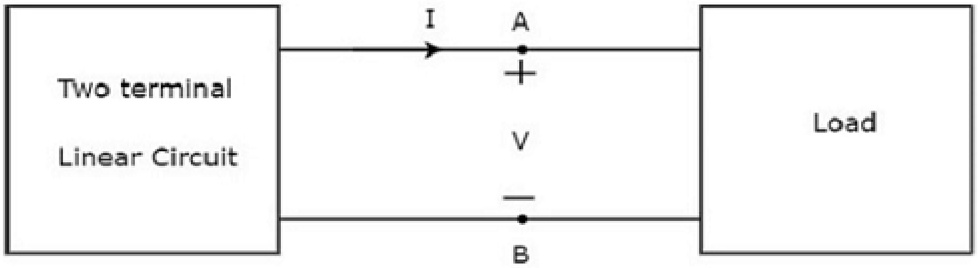
Thevenin's Theorem :

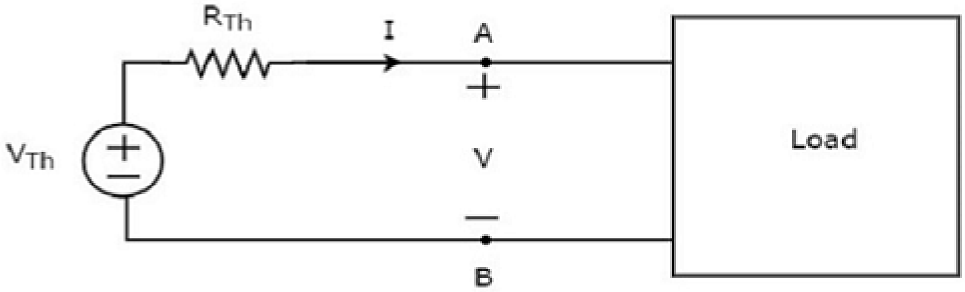
STATEMENT : It states that any two terminal linear network or circuit can be represented with an equivalent network or circuit, which consists of a voltage source in series with a resistor. It is known as Thevenin's equivalent circuit. A linear circuit may contain independent sources, dependent sources, and resistors.

If the circuit contains multiple independent sources, dependent sources, and resistors, then the response in an element can be easily found by replacing the entire network to the left of that element with a **Thevenin's equivalent circuit**.

The **response in an element** can be the voltage across that element, current flowing through that element, or power dissipated across that element.

This concept is illustrated in following figures.





Thevenin's equivalent circuit resembles a practical voltage source. Hence, it has a voltage source in series with a resistor.

- The voltage source present in the Thevenin's equivalent circuit is called as Thevenin's equivalent voltage or simply **Thevenin's voltage, V_{Th}.**
- The resistor present in the Thevenin's equivalent circuit is called as Thevenin's equivalent resistor or simply **Thevenin's resistor, R_{Th}.**

Methods of Finding Thevenin's Equivalent Circuit

There are three methods for finding a Thevenin's equivalent circuit. Based on the **type of sources** that are present in the network, we can choose one of these three methods. Now, let us discuss two methods one by one. We will discuss the third method in the next chapter.

Method 1

Follow these steps in order to find the Thevenin's equivalent circuit, when only the **sources of independent type** are present.

- **Step 1** − Consider the circuit diagram by opening the terminals with respect to which the Thevenin's equivalent circuit is to be found.
- **Step 2** − Find Thevenin's voltage **V_{Th}** across the open terminals of the above circuit.
- **Step 3** − Find Thevenin's resistance **R_{Th}** across the open terminals of the above circuit by eliminating the independent sources present in it.
- **Step 4** − Draw the **Thevenin's equivalent circuit** by connecting a Thevenin's voltage V_{Th} in series with a Thevenin's resistance R_{Th}.

Maximum power transfer theorem :

It states that the DC voltage source will deliver maximum power to the variable load resistor only when the load resistance is equal to the source resistance.